

Hailstone Terminal Velocity

$$\begin{aligned}
 r &:= \frac{.005}{2} & g &:= 9.8 & d &:= 2 \cdot r & v &:= .0000133 \\
 \gamma_{\text{hail}} &:= 6000 & p &:= 96000 & T &:= 273 \\
 \rho_{\text{air}} &:= \frac{p}{287 \cdot T} & \rho_{\text{air}} &= 1.2253
 \end{aligned}$$

(SI units)

Use correlation from Frank White's book.

$$p(x) := \sqrt[3]{8 \cdot \gamma_{\text{hail}} \cdot \frac{r}{3 \cdot \rho_{\text{air}} \cdot \left(12 \cdot \frac{v}{r \cdot x} + \frac{6}{1 + \sqrt{2 \cdot r \cdot \frac{x}{v}}} + 0.4 \right)}} - x$$

$x := 10$ x is the "guess" for the root solver

$V := \text{root}(p(x), x)$ V is the terminal velocity in meters/second

$$V = 7.9559 \quad \text{Re} := d \cdot \frac{V}{\nu} \quad \text{Re} = 2.991 \times 10^3$$

$$C_D := \frac{24}{\text{Re}} + \frac{6}{1 + \sqrt{\text{Re}}} + 0.4 \quad C_D = 0.5158$$

Now a program is written to calculate the terminal velocity over a range of hailstone diameter ranging from .1cm to 15 cm.

$$w(x, r1) := \sqrt[3]{8 \cdot \gamma_{\text{hail}} \cdot \frac{r1}{3 \cdot \rho_{\text{air}} \cdot \left(12 \cdot \frac{v}{r1 \cdot x} + \frac{6}{1 + \sqrt{2 \cdot r1 \cdot \frac{x}{v}}} + 0.4 \right)}} - x$$

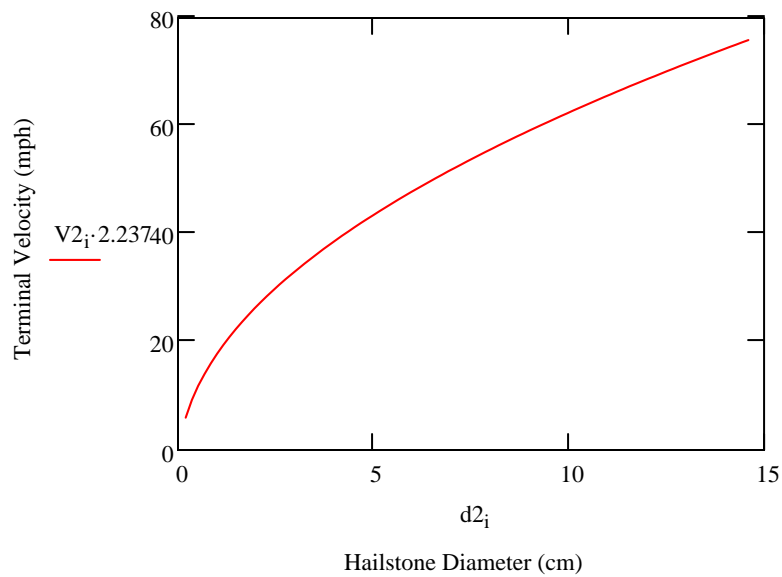
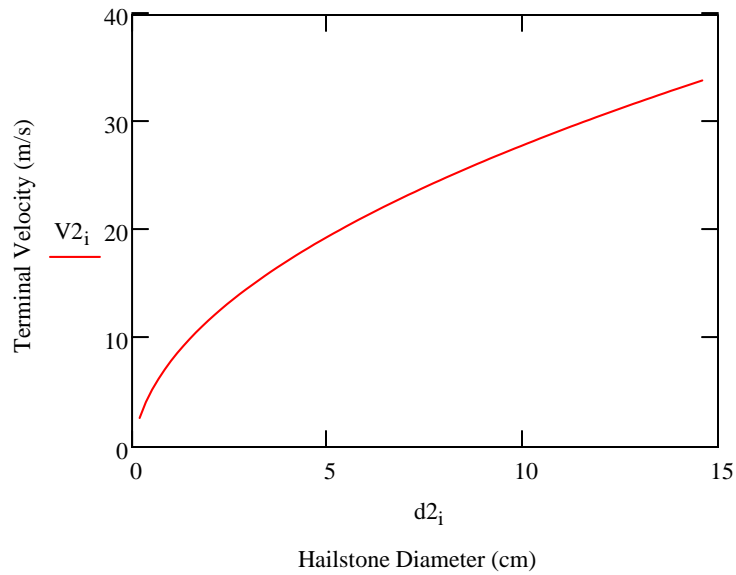
$V1(x, r1) := \text{root}(w(x, r1), x)$ $V1$ is a function - where x is the guess and $r1$ is the hailstone diameter.

$$V2 := \begin{cases} \text{for } k \in 0..90 \\ \left| \begin{array}{l} r2 \leftarrow \frac{.001 + k \cdot .0008}{2} \\ V2_k \leftarrow V1(5, r2) \end{array} \right. \\ V2 \end{cases}$$

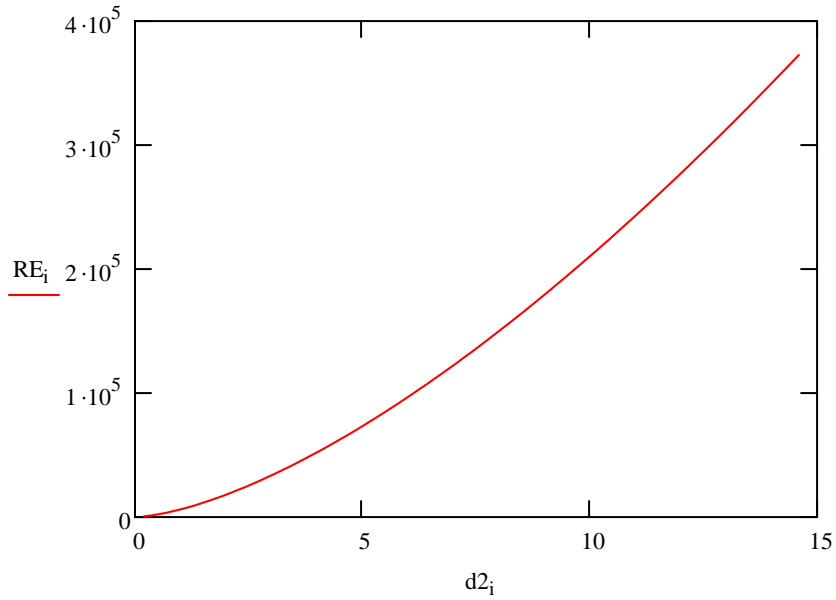
A for loop is used to calculate velocity for 90 hailstone diameters. The guess for each call of function V1 is 5.

$i := 0..90$ $d2_i := (0.001 + i \cdot .0008) \cdot 200$ Diameters in centimeters.

Now graph the terminal velocity and Reynolds number



$$RE_i := d2_i \cdot 0.01 \cdot \frac{V2_i}{\nu}$$



$$C_{D_i} := \frac{24}{RE_i} + \frac{6}{1 + \sqrt{RE_i}} + 0.4$$

